1.6 Piecewise Functions

A Piecewise-Defined Functions
A piecewise-defined function requires more than one formula to explicitly define the function. Each formula is defined on a different interval.

Note: The domain of the piecewise-defined function is the union of all intervals used to define the function.

Ex 1. Let consider the function $y = f(x) = |x - 2|$.
Write this function as a piecewise function and then graph it.

Ex 2. The Heaviside function is defined by:

$$H(x) = \begin{cases} 1, & if\ x \geq 0 \\ 0, & if\ x < 0 \end{cases}$$

Graph this function and analyze its continuity.

Ex 3. The Greatest Integer Functions $[x]$ defined in words by: "$f(x)$ is te greatest integer n that is less or equal to x" or:

$$[x] = n, \quad n \in Z, \quad n \leq x < n + 1$$

Graph this function and analyze its continuity.

Ex 4. The XYZ function is defined by:

$$f(x) = \begin{cases} 1, & if\ x\ is\ rational \\ 0, & if\ x\ is\ irrational \end{cases}$$

Can be this function represented graphically? How?

Ex 5. Graph the following piecewise function:

$$f(x) = \begin{cases} x - 4, & if\ x > 2 \\ -1, & if\ x = 2 \\ x^2 - 4, & if\ x < 2 \end{cases}$$

Ex 6. Let consider the function:

$$y = f(x) = x - |x+1| - |x+2|$$

Write this function as a piecewise function and then graph it.

Ex 7. Write the function $f(x) = \sqrt{x^2}$ as a piecewise function and then graph it.

Ex 8. Write the function $f(x) = \frac{|x|}{x}$ as a piecewise function and then graph it.

B Continuity

The graph of a continuous function can be drawn "without lifting pencil from paper".

A continuous function has no holes, finite gaps (jumps), or infinite breaks.

Ex 9. For what value of the constant c is the function

$$f(x) = \begin{cases} x + c & if\ x < 2 \\ cx^2 + 1 & if\ x \geq 2 \end{cases}$$

continuous at every number?

C Absolute Value of a Function

The absolute value of a function is defined by:

$$|f(x)| = \begin{cases} f(x), & if\ f(x) \geq 0 \\ -f(x), & if\ f(x) < 0 \end{cases}$$

Ex 10. For each case, graph the original function $f(x)$ and then graph $|f(x)|$.

a) $f(x) = 2x - 6$

b) $f(x) = x^2 - x - 6$

Reading: Nelson Textbook, Pages 46-51
Homework: Nelson Textbook, Page 51: #1cf, 3, 5cd, 7, 8, 11, 14, 15

